A PRIORITY INVENTORY MODEL OF THE MILITARY SUPPLY SYSTEM

by

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THESIS

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ABSTRACT

Very few inventory models of the military supply system have been developed that explicitly take into account the effects of the priority system. The priority inventory model of the military supply system is formulated to minimize the sum of ordering and holding costs subject to constraints on the number of backorders allowable by issue priority group. A minimum cost solution of a typical inventory problem of this nature is obtained by computer simulation.

TABLE OF CONTENTS

4

LIST OF TABLES

I. INTRODUCTION

In recent years, many inventory models have been developed and implemented successfully. Much attention has been given to the problem of applying scientific inventory procedures to the military supply system, due to the high dollar value of the inventory and the importance of having an effective supply system to support the military services. However, at least one important aspect of the military supply system has been largely ignored. That aspect is the effect of the priority system on the performance of the military supply system. This thesis derives a model of the military supply system which accounts for the different priorities placed on supply transactions.

Issue priority designators of one (highest priority) to 20 (lowest priority) are used to denote the priority of a military supply transaction. However, customers within the military supply system do not arbitrarily place priorities on their supply requisitions. The issue priority designator of a particular supply transaction is determined by the following two items:

1. the force/activity designator of the requisitioner, and
2. the urgency of need designator of the item being requisitioned.

The force/activity designator (FAD) of a unit ranges from FAD I (highest) to FAD V (lowest) and is assigned by the major commander or the Joint Chiefs of Staff. Basically, a unit's force/activity designator

is assigned according to its relative readiness for combat or deployment. Below is a summary of each force/activity designator level and its general description.

(1) FAD I -- assigned to units engaged in a general war (such as combat units in World War II).

(2) FAD II -- assigned to units engaged in active combat short of a general war or those units immediately available for combat operations upon outbreak of hostilities (such as combat units in Vietnam or air defense missile units in the United States).

(3) FAD III -- assigned to those units being maintained at a level of operational readiness such that they could be deployed by D + 30 days (D-day being the day of outbreak of hostilities).

(4) FAD IV -- assigned to those units to be deployed after D + 30 days.

(5) FAD V -- assigned to those units to be deployed subsequent to D + 90 days and all other forces and activities not otherwise designated.

The urgency of need designator of a unit ranges from designator A (highest) to designator D (lowest). Basically, the urgency of need designator of a supply transaction is "determined by the essentiality of the materiel being requisitioned to the accomplishment of the military mission assigned to the force/activity" [Ref. 1, p. 2-2]. Below is a summary of the criteria for the assignment of urgency of need designators to supply transactions.

(1) Designator A -- used for requisitioning materiel without which the unit is unable to perform its assigned missions or tasks.

(2) Designator B -- used for requisitioning materiel which impairs the capability of a unit to perform its assigned missions or tasks.

(3) Designator C -- used for requisitioning materiel needed on a more urgent basis than routine.

(4) Designator D -- used for requisitioning materiel on a routine basis or initial filling of allowances.

Together, the force/activity designator assigned to a unit and the urgency of need designator for a particular supply transaction determine the issue priority designator of that particular requisition. The correspondence between force/activity designators I through V, urgency of need designators A through D, and issue priority designators one through 20 is illustrated in Table I.

Urgency of need designator (UND)	Force/activity designator (FAD) I	II	III	IV	V
UND A	01	02	03	07	08
UND B	04	05	06*	09	10
UND C	11	12	13	14	15
UND D	16	17	18	19	20

*For example, an issue priority designator of 06 would be assigned a supply transaction for a unit with force/activity designator III and urgency of need designator B.

TABLE I -- Issue Priority Designators

II. THE PRIORITY INVENTORY MODEL

In order to be as realistic as possible, current operating policies for the Army's inventory system were used where applicable. In particular, the inventory operating policies of the Sixth U. S. Army Stock Control Center, located at the Presidio of San Francisco, were incorporated into the model wherever possible. In addition, most of the input parameters for the computer simulation were also provided by the Sixth U. S. Army Stock Control Center.

Basically, the priority inventory model described in this section is similar to a lot size-reorder point model [Ref. 2, p. 159-167] with one significant feature added to account for different priorities of requisitions within the Army supply system. (Although this thesis dealt primarily with the Army supply system, much of it will be applicable to the other military services as most of the procedures cited here are Army implementations of Department of Defense procedures that are applicable to all the services.)

A. ASSUMPTIONS OF THE MODEL

The following assumptions were used in formulating this priority inventory model of the military supply system.

(1) Issue priority groups rather than issue priority designators were assigned to each supply transaction.

(2) All demands were for a quantity of one each.

(3) Demands per unit time had a Poisson distribution for each item in the inventory.

(4) The standard economic order quantity [Ref. 3, p. 93-95] was used as the reorder quantity for each item in the inventory.

(5) Fixed costs were ignored.

(6) The only variable costs considered were ordering costs ($7.50 per order) and holding costs (20% of each item's unit cost per year) [Ref. 4, p. 39] .

(7) All requisitions were either filled or backordered (i.e., no lost sales).

(8) Order-ship times were normally distributed.

(9) The probability of the occurrence of each issue priority group was the same for all items in the inventory.

B. DISCUSSION OF THE ASSUMPTIONS

Although there are 20 different issue priority designators within the military supply system, for most practical purposes they are grouped into four issue priority groups as follows:

Issue Priority Group	Issue Priority Designators
One	01 through 03
Two	04 through 08
Three	09 through 15
Four	16 through 20

TABLE II -- Issue Priority Groups

There were two reasons for using issue priority groups rather than issue priority designators in this model.

(1) Within the Army supply system, requisitions are handled by issue priority group, rather than issue priority designator, for most filling, transportation, and shipping purposes.

(2) Data was available from the Sixth U. S. Army Stock Control Center on supply transactions by issue priority group but not by priority designator.

For the rest of this thesis, whenever the priority of a requisition is mentioned, it refers to the issue priority group of the requisition and not the issue priority designator.

While the allowing of more than one item to be requisitioned at a time would have been a more general situation, it was felt that this would have detracted from the main purpose of the model, which was to study the effect of the priority system. Relatively minor changes could have been made to this priority inventory model to incorporate the possibility of multiple units being demanded in a single supply transaction (Appendix D).

The assumption of Poisson demand distribution is a common one in inventory models and it seemed to fit the historical demand data better than any other demand distribution. No Chi-Square goodness-of-fit tests were run on the data, as only 13 months of demand history was available from the Sixth U. S. Army Stock Control Center. Generally, there must be an expected value of at least 5 in each separate classification in order for the Chi-Square statistic to be accurate in the goodness-of-fit test [Ref. 5, p. 207]. In addition, the assumption of Poisson demand distribution led to time between demands being

exponentially distributed, which was then used as a basis for the computer simulation of the priority inventory model.

If the lot size-reorder point model [Ref. 2, p. 159-167] had been used, the economic order quantity would have been the reorder quantity that would have minimized variable costs, subject to constraints on backorders. However, since the priority inventory model, as formulated in the next section, incorporated many of the features of the lot size-reorder point model and it has been shown that variable costs are relatively insensitive to small changes in the quantity ordered [Ref. 3, p. 96-99], it was decided to reorder the economic order quantity where applicable.

It is standard procedure to ignore fixed costs in the analysis of inventory systems (as they are independent of the inventory policy used) and the minimization of variable costs is usually the objective in these inventory models. In actuality, it is very hard to break down the costs of operating an inventory system into separate categories. In fact, it is often nearly impossible to account for the total costs of an inventory system, much less break down the cost into components such as ordering cost, holding cost, backorder cost, etc. Reference 3, pages 77-86, contains a good discussion of the types of costs pertinent to military inventory systems.

In this priority inventory model, ordering cost of $7.50 per order and inventory holding rate of 20% per year were used for two reasons:

(1) They were realistic in the sense that at least one Army inventory system is using them (Sixth U. S. Army Stock Control Center).

(2) They were the only semi-official figures available.

It is even harder to get a reasonable estimate of backorder cost than it is for ordering cost and holding cost. Backorder costs have sometimes been expressed in terms of a "goodwill cost" for backordering an item when the inventory system is out of stock. However, this did not seem to have much meaning in a military context. This priority inventory model did not have an explicit backorder cost, but rather put constraints on the number of backorders allowable by issue priority group. This seems to be more applicable to a military situation, as it allows the model to take into account the implied essentiality of the different issue priority groups.

The meaning of lost sales in a military context is, at best, very questionable. While it is true that units sometime have to cancel requisitions (when they are deactivated or reduced in size, for example), it was assumed that the number of cancellations (i.e., "lost sales") was small and could be ignored, thus leading to the assumption that all requisitions were either filled or backordered.

The assumption that the order-ship times were normally distributed was made only to be able to explicitly formulate the model and solve typical inventory problems using it. The only data available from the Sixth U. S. Army Stock Control Center was the mean and variance of the order-ship time distribution with no indication of the actual underlying

distribution. If other data would become available for order-ship times, the appropriate distribution should be substituted for the normal distribution.

In order to fully implement this model, it would be necessary to collect and maintain data on the number of requisitions by priority and line item, which is not now being done. The only data available from the Sixth U. S. Army Stock Control Center was consolidated data on the total number of requisitions by issue priority group. Since it was not broken down by individual line item and issue priority designator, it was necessary to make the last assumption.

It should be pointed out that this priority inventory model is most applicable to consumable (i.e., non-repairable) secondary items and/or repair parts with relatively low unit cost. These types of items in an inventory system are particularly well suited to being managed utilizing a maximum of automatic data processing equipment and a minimum of human effort. The Sixth U. S. Army Stock Control Center is a prototype of a mechanized stock control center that may eventually control most of the Army inventory system within the continental United States (and perhaps overseas with some modifications). This model could be easily adapted for their use in the control of low cost consumable supplies and/or repair parts.

C. FORMULATION OF THE MODEL

As previously stated, this priority inventory model was basically a lot size-reorder point model with three variable levels added besides the reorder point. Reference 6, pages 1-20, derives a similar model which uses implied weights for each issue priority group to establish optimum reserve levels. The three major differences between that model and the priority inventory model, as formulated in this section, were as follows:

(1) The reserve level model used a fixed lead time, while the priority inventory model used random lead times that were normally distributed.

(2) The reserve level model was periodic review, while the priority inventory model was continuous review.

(3) The reserve level model used implied weights as a measure of the importance of high priority demands, while the priority inventory model constrained the number of backorders allowable for each issue priority group.

The three variable levels and the reorder point will be called inventory operating levels for the rest of this thesis.

The objective of the priority inventory model was to minimize total variable costs (ordering and holding costs) subject to constraints on the number of backorders allowable for each of the four issue priority groups. The decision variables were the four inventory operating levels described below:

(1) Inventory operating level one (reorder point) --based on inventory position (the quantity on hand plus the quantity on order minus the quantity backordered for each item). The economic order quantity was reordered whenever the inventory position of an item was reduced to level one.

(2) Inventory operating level two -- based on the on hand quantity. When on hand quantity of an item was reduced to level two, further requisitions would only be filled for those items with priorities 1, 2, or 3, while requisitions for the item having a priority of 4 were backordered.

(3) Inventory operating level three -- based on the on hand quantity. When on hand inventory was reduced to level three, priority 1 and 2 requisitions were filled, while priority 3 and 4 requisitions were backordered.

(4) Inventory operating level four -- based on the on hand quantity. When the on hand quantity was reduced to level four, only priority 1 requisitions were filled and all others were backordered.

Thus, as on hand inventory was being lowered, only successively higher priority requisitions were being filled and the lower priority requisitions were being backordered. When replenishment stock was received, then normal filling of requisitions was resumed until the on hand quantity again fell to one of the inventory operating levels.

In general, inventory operating level one (the reorder point) could be either greater than, equal to, or less than any of the other levels, since it was based on inventory position. The reorder point could be positive, negative, or zero while the other inventory operating levels were restricted to be greater than or equal to zero, as they were based on the on hand quantity of each item in the inventory.

For example, assume that the status of the inventory was as follows for one item:

(1) Level one (reorder point) = -1.

(2) Level two = 0.

(3) Level three = 0.

(4) Level four = +1.

(5) On hand quantity was 1.

(6) No outstanding orders or backorders (i.e., inventory position was also equal to one).

If a low priority requisition was then received, it would be backordered, as the on hand quantity and level four both being equal to one would suspend the filling of all requisitions except those with a priority of one. Thus, any two demands, regardless of their priority, would cause the inventory position to fall to the reorder point, whether they were filled or backordered.

The following relationships were true for inventory operating levels two, three, and four:

(1) Levels two and three positive $\Longrightarrow$ level two $\geq$ level three.

(2) Levels two and four positive $\Longrightarrow$ level two $\geq$ level four.

(3) Levels three and four positive $\Longrightarrow$ level three $\geq$ level four.

(4) Levels two, three and four positive $\Longrightarrow$ level two $\geq$ level three $\geq$ level four.

Inventory problems resulting from this priority inventory model (minimize ordering cost plus holding cost subject to constraints on the percentage of backorders allowable by issue priority group) did not appear to have any analytical solution, even when the economic order quantity was specified as the reorder quantity, as the four inventory operating levels still remained as unknowns. The M.I.T. Non-Reparables Model [Ref. 7, p. 24-29] used by the Army minimizes the same two costs, but there is only one constraint, that being a constraint

on national availability. The priority inventory model, as formulated in this section, puts constraints on availability of each item by issue priority group, rather than just one constraint on overall availability. In addition, the M.I.T. model assumes a constant lead time and a normal distribution of lead time demand.

General inventory theory and the lot size-reorder point model suggested that these levels should be as low as possible without violating the constraints on the backorders allowable. The lack of an analytical solution to this problem and the availability of data on the parameters of a large number of typical inventory items (low cost consumables and/or repair parts) led to a computer simulation of this priority inventory model as the method used to obtain the minimum cost solution to constrained inventory problems of this nature.

III. COMPUTER SIMULATION OF THE PRIORITY INVENTORY MODEL

The computer simulation of this priority inventory model was programmed in PL/1 and run on the IBM 360/67 computer at the Naval Postgraduate School. The computer program and samples of the computer output are at the end of this thesis. In addition, a detailed flow chart of the main portion of the program (lines 115 through 220) is included as Appendix A and a list of the variables and what they represented is included as Appendix B.

The computer program used Monte Carlo techniques as the basis for the simulation of the occurrence of random events that had known or assumed probability distributions. The multiplicative congruential method [Ref. 8, p. 51-52] was used to generate pseudorandom numbers on the unit interval (lines 19 through 29 of the computer program), which were in turn used to generate other random functions (lines 35 through 70). Appendix C contains the results of Chi-Square goodness-of-fit tests [Ref. 5, p. 201-202] for samples of pseudorandom numbers for different random number seeds.

The actual program used for the simulation of the priority inventory model is contained on lines 1 through 299 of the sample program at the end of this thesis. Lines 300 through 312 were the input data for a particular set of simulation runs. Basically, lines 117 through 175 handled an order of the $i^{\underline{th}}$ item arriving on the $j^{\underline{th}}$ day, while lines

174 through 218 handled demands for the $i^{\underline{th}}$ item on the $j^{\underline{th}}$ day. Appendix A contains a detailed flow chart of lines 115 through 220, the main portion of the simulation program.

The program executed the simulation of the priority inventory model for as many as 200 different sets of inventory operating levels (more sets of data could have been run by changing the 200 on line 86 to the appropriate number and including more sets of data as input to the program). The simulation was arbitrarily set to run for 10 years and each set of inventory operating levels required approximately $5\frac{1}{2}$ seconds of computer time and the results were printed out on a separate page of the output. Proportionate decreases (or increases) in computer time could have been affected by decreasing (or increasing) the 10 years of simulated operation of the supply system. However, it was felt that at least 10 years of simulated operation was necessary in order to approximate a steady-state and to negate the effects of arbitrarily setting the beginning on hand inventory levels of each item to 20. Twenty was chosen so that all items would be above their reorder point when the simulation started.

Appendix D contains a detailed discussion of the possible further uses of the computer simulation of this priority inventory model.

IV. SOLUTION OF A SAMPLE PROBLEM

Since constrained inventory problems resulting from this priority inventory model did not appear to have analytical solutions, the computer simulation was used to obtain inventory operating levels that resulted in minimum costs in a typical problem of this nature. The sample problem that was solved had four items in the simulated inventory system (in effect, four one-item problems were solved, as the model treated each item separately) and was formulated as follows:

Minimize ordering costs + holding costs (for each item)

Subject to
(1) Probability of a backorder for a priority one requisition $\leq .01$.
(2) Probability of a backorder for a priority two requisition $\leq .05$.
(3) Probability of a backorder for a priority three requisition $\leq .10$.
(4) Probability of a backorder for a priority four requisition $\leq .20$.

The parameters for each item were obtained from the Sixth U. S. Army Stock Control Center and are included as Appendix E.

The following steps were followed in obtaining the minimum cost solution to the above problem:

(1) All the inventory operating levels were set equal to zero for all items.

(2) The approximate mean lead time demand was calculated by multiplying the mean lead time by the mean demand rate for each item. For this problem, the approximate mean lead time demand for items 1, 2, 3, and 4 respectively was 4, 5, 3, and 1 units.

(3) For each item, inventory operating level one (the reorder point) was set to 20 less than the approximate mean lead time demand calculated above (all the other levels remaining at zero). The simulation was then run for all integer reorder points from 20 less than to 15 greater than the approximate mean lead time demand (other problems might require different ranges in this step).

(4) The initial solution was obtained from the printout in step (3) above (each printout gave the number of backorders by priority and item for the simulated 10 years of operation for each different set of inventory operating levels) by finding the lowest reorder point that satisfied all the constraints for each item. The initial solution for this problem was as follows (inventory operating levels two, three, and four being zero):

	Reorder Point	Cost
Item 1	6	\$355.15
Item 2	2	\$ 38.59
Item 3	4	\$659.58
Item 4	1	\$107.28

Total Cost = \$1160.60

(5) For each item, a lower bound on the reorder point was established by finding the lowest reorder point such that some redistribution of backorders could have satisfied all the constraints (obtained from the printout in step (3) above). For this problem, the lower bounds were found to be 2, -15, 1, and -1 respectively for items 1, 2, 3, and 4.

(6) For each item, the reorder point (inventory operating level one) was set equal to its lower bound and inventory operating levels two, three, and four (or any combination of them) were raised to try to redistribute the backorders to meet the constraints. In raising inventory operating levels two, three and four, the following rules-of-thumb were used:

(a) Raising inventory operating level two would decrease priority 1, 2, and 3 backorders and increase priority 4 backorders.

(b) Raising inventory operating level three would decrease priority 1 and 2 backorders and increase priority 3 and 4 backorders.

(c) Raising inventory operating level four would decrease priority 1 backorders and increase priority 2, 3, and 4 backorders.

(7) For item(s) that had no redistribution that would satisfy the constraints when the reorder point was at its lower bound, the reorder point was raised one unit and inventory operating levels two, three, and four were raised as in step (6). For this problem, three out of the four items had minimum cost in step (6) and only one had to go to step (7) for a minimum cost solution (for other problems of this nature, the reorder point might have to be raised still further to obtain the minimum cost solution that was feasible). Since raising the reorder point decreased backorders for all four priorities, but also raised the cost of all four items, the lowest reorder point satisfying all the constraints provided the minimum cost solution for each item. The following was the minimum cost solution obtained for this problem from the computer simulation of the priority inventory model:

Inventory Operating Levels

	1	2	3	4	Cost
Item 1	2	2	1	0	$279.57
Item 2	-15	4	1	0	$ 28.93
Item 3	2	4	1	0	$601.99
Item 4	1	3	1	0	$ 81.92

Total Cost = $992.41

Virtually all of the savings in inventory costs were due to reduced average on hand quantity. The following was the reduction in average on hand quantity obtained from the initial to the final solution:

(1) Item 1 -- from 12.5 to 8.8.

(2) Item 2 -- from 160.7 to 139.3.

(3) Item 3 -- from 14.8 to 13.3.

(4) Item 4 -- from 17.8 to 15.7.

Since the economic order quantity was used in each instance and all requisitions had to be eventually filled, the number of orders placed for each item was nearly the same for each computer run, varying only slightly due to the stochastic nature of the simulation.

Since it was not immediately obvious that the economic order quantity would result in "optimal" solutions, different reorder quantities 10% and 20% both above and below the standard economic order quantity (EOQ) were used in the simulation. The final solution for each different set of reorder quantities is given below:

	-20%	-10%	EOQ	+10%	+20%
Item 1	$325.03	$295.20	$279.57	$297.82	$289.75
Item 2	$ 33.52	$ 35.36	$ 28.93	$ 29.82	$ 32.91
Item 3	$653.46	$618.85	$601.99	$581.15	$606.64
Item 4	$ 92.16	$ 93.63	$ 81.92	$ 95.59	$ 86.89
Total Cost	$1104.17	$1043.04	$992.41	$1004.38	$1016.19

While this did not prove that the economic order quantity was optimal, it suggested that it was close to the optimal reorder quantity and seemed to justify its assumption as the reorder quantity that would minimize costs subject to backorder constraints.

V. CONCLUSIONS

The following were the general conclusions drawn:

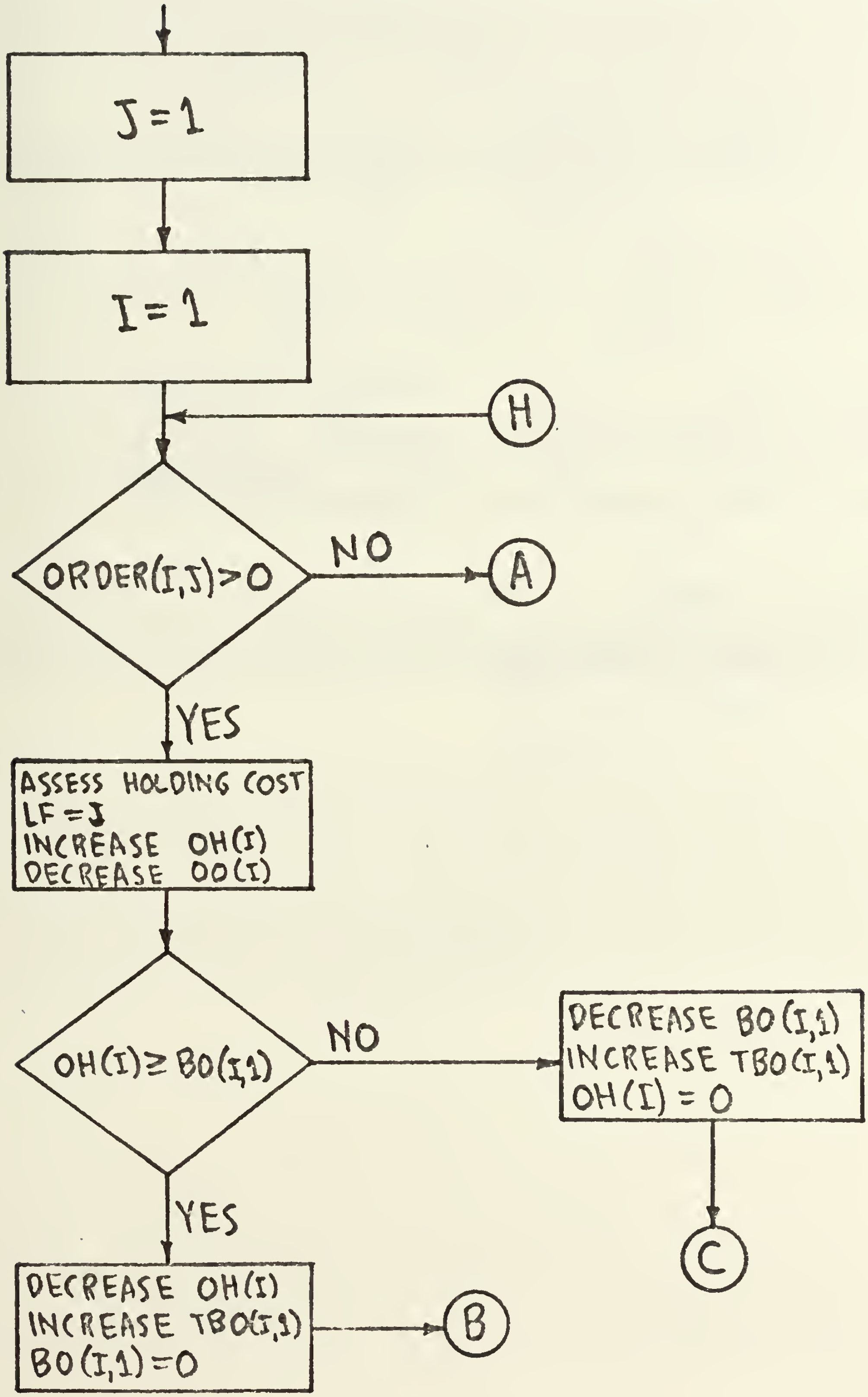
(1) This priority inventory model takes into account the implied essentiality of higher priority requisitions by filling only successively higher priority demands as on hand inventory is lowered.

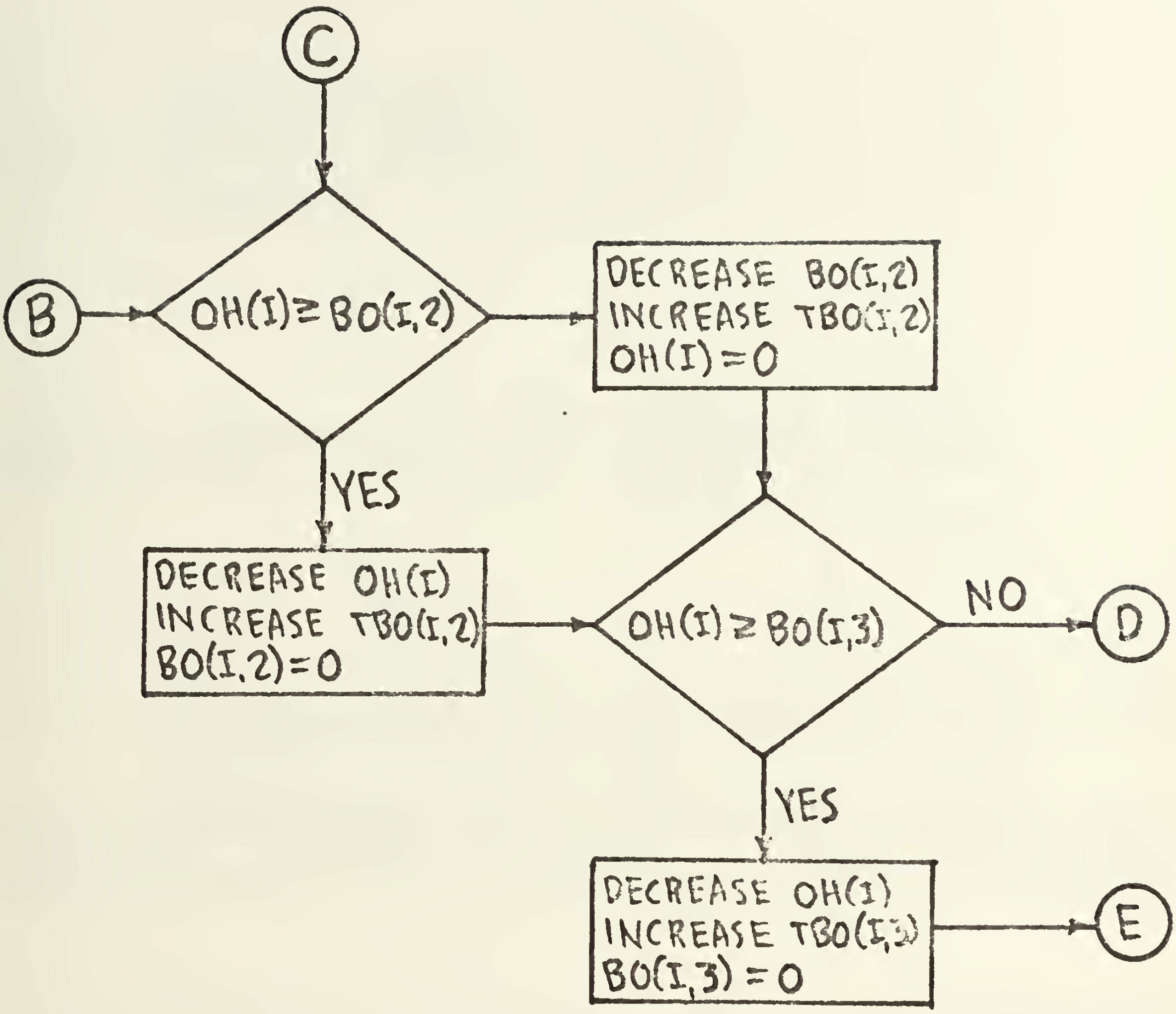
(2) Constraints on the number of backorders allowable by issue priority group seem to fit the original purpose of the military priority system, as it concentrates backorders in the lower issue priority groups where they have less of an impact.

(3) This priority inventory model is particularly well suited for adoption by a mechanized supply system, as a computer could be programmed to fill or backorder a requisition depending on the inventory operating levels and on hand quantity of an item.

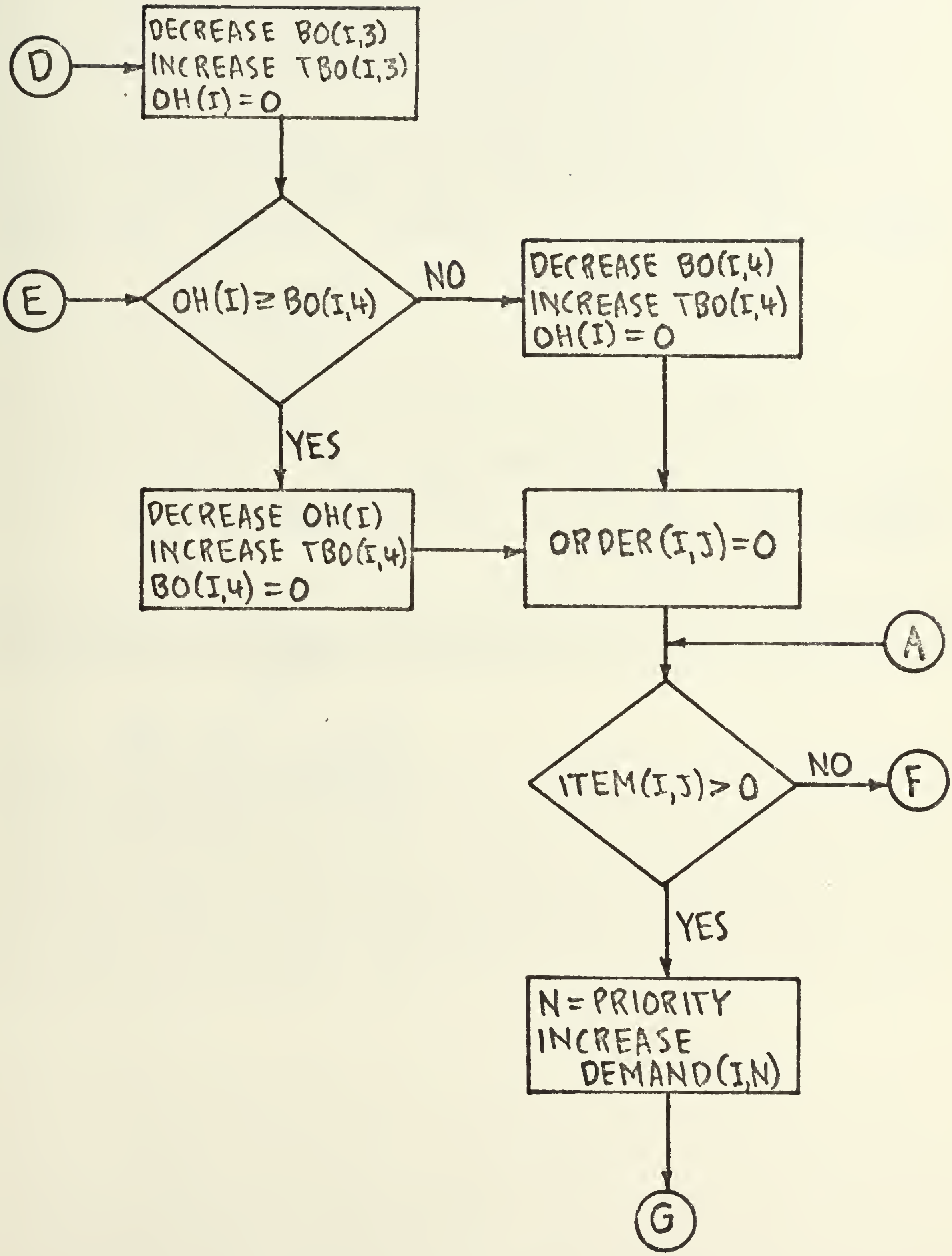
APPENDIX A

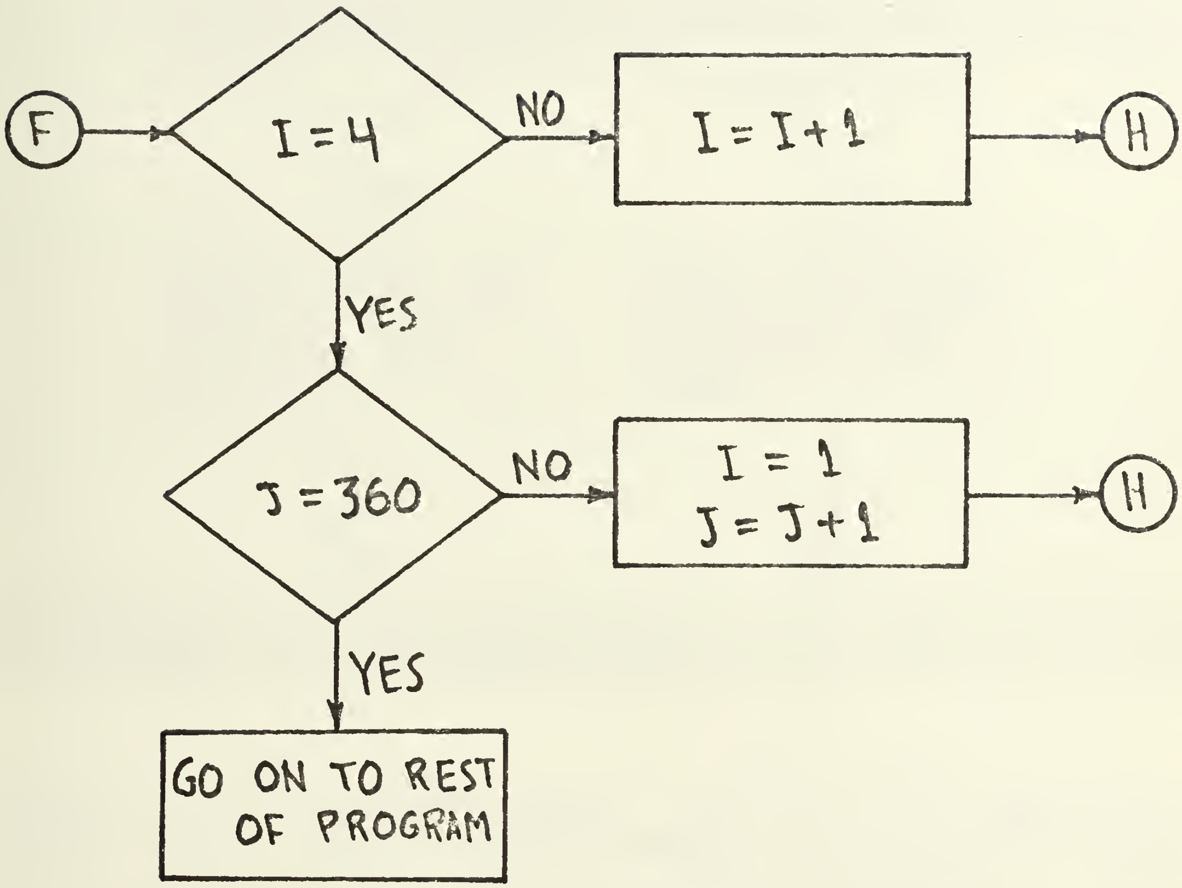
FLOW CHART OF THE SIMULATION

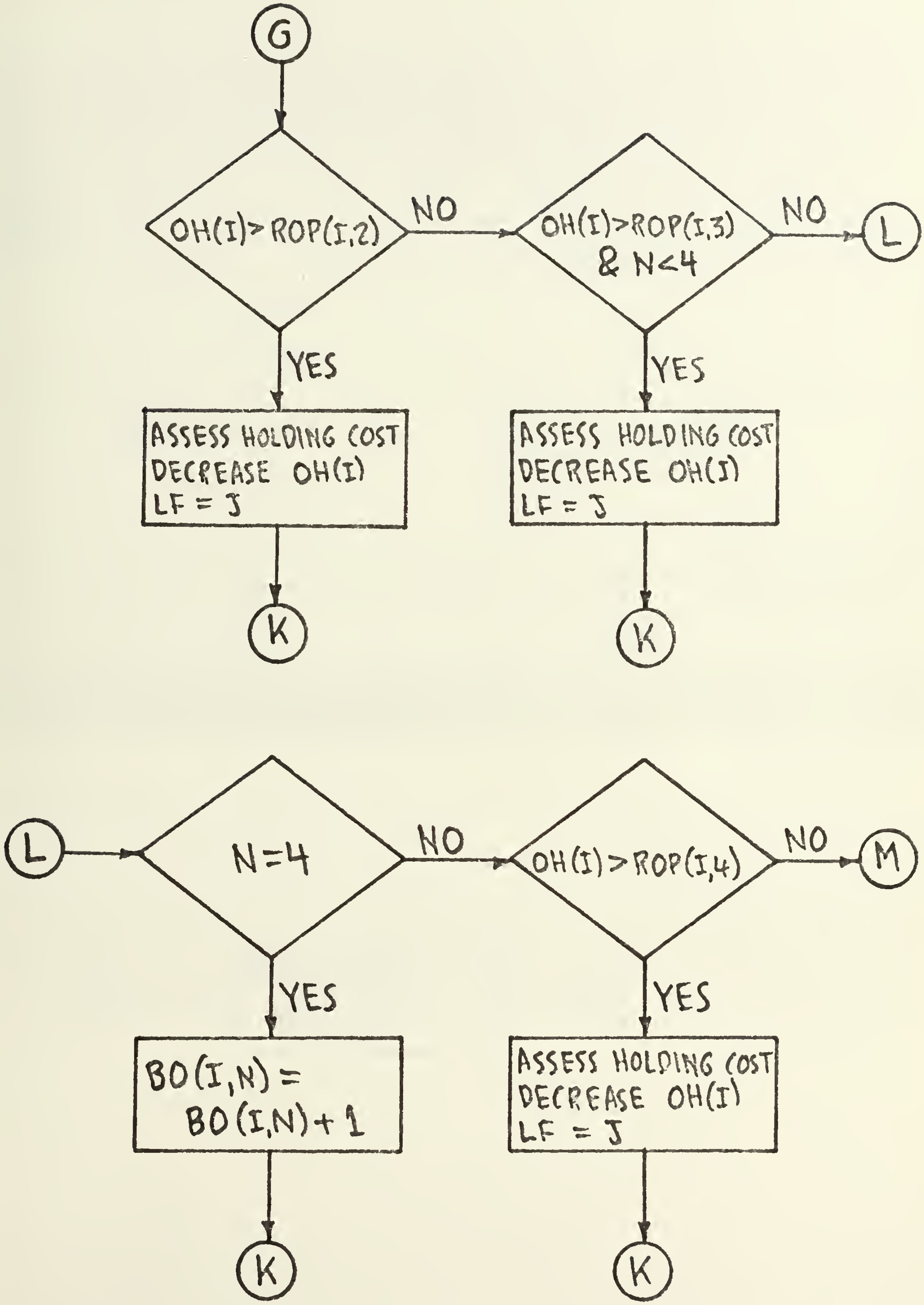


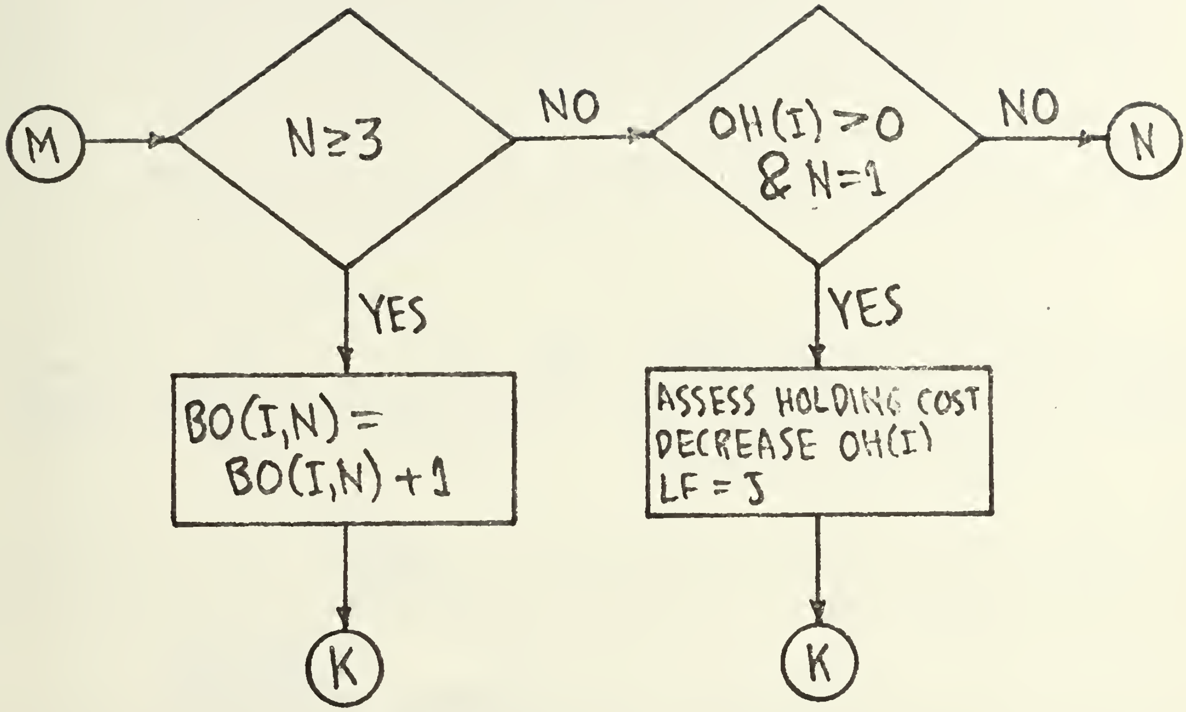


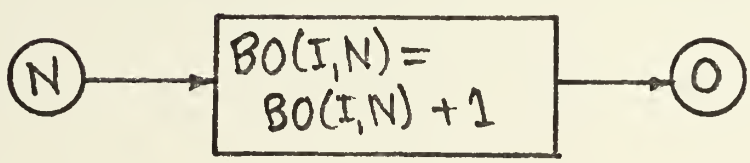
C
B
OH(I) ≥ BO(I,2)
DECREASE BO(I,2)
INCREASE TBO(I,2)
OH(I) = 0
YES
DECREASE OH(I)
INCREASE TBO(I,2)
BO(I,2) = 0
OH(I) ≥ BO(I,3)
NO
D
YES
DECREASE OH(I)
INCREASE TBO(I,3)
BO(I,3) = 0
E

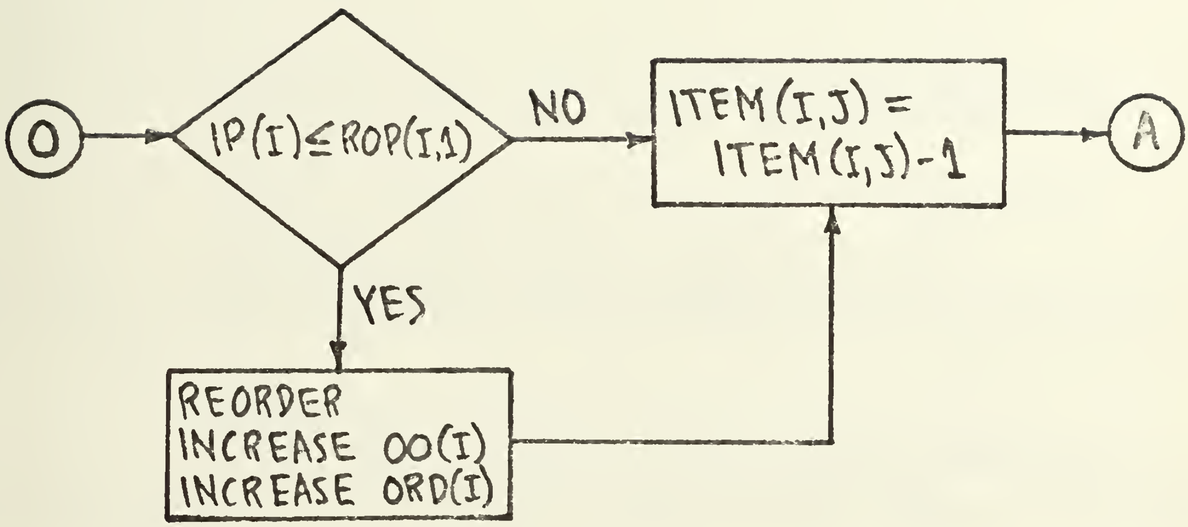

D
DECREASE BO(I,3)
INCREASE TBO(I,3)
OH(I) = 0
E
OH(I) ≥ BO(I,4)
NO
DECREASE BO(I,4)
INCREASE TBO(I,4)
OH(I) = 0
YES
DECREASE OH(I)
INCREASE TBO(I,4)
BO(I,4) = 0
ORDER(I,J) = 0
A
ITEM(I,J) > 0
NO
F
YES
N = PRIORITY
INCREASE
DEMAND(I,N)
G


F
I=4
NO
I=I+1
H
YES
J=360
NO
I=1
J=J+1
H
YES
GO ON TO REST OF PROGRAM


G
OH(I)>ROP(I,2)
NO
OH(I)>ROP(I,3) & N<4
NO
L
YES
ASSESS HOLDING COST
DECREASE OH(I)
LF = J
YES
ASSESS HOLDING COST
DECREASE OH(I)
LF = J
K
K
L
N=4
NO
OH(I)>ROP(I,4)
NO
M
YES
BO(I,N) = BO(I,N)+1
YES
ASSESS HOLDING COST
DECREASE OH(I)
LF = J
K
K


M
N≥3
NO
OH(I)>0
& N=1
NO
N
YES
BO(I,N) =
BO(I,N) + 1
YES
ASSESS HOLDING COST
DECREASE OH(I)
LF = J
K
K


N
BO(I,N) =
BO(I,N) + 1
O



O
IP(I)≤ROP(I,1)
NO
ITEM(I,J) =
ITEM(I,J)-1
A
YES
REORDER
INCREASE OO(I)
INCREASE ORD(I)

APPENDIX B

IDENTIFICATION OF VARIABLES USED IN THE PROGRAM

The following is a list of the variables and arrays used in the computer simulation program and what they were used for or represented, as applicable.

(1) A, B, X, Y, Z, H, I, J, K, L, N, COST, and SEED were variables used for calculations, in do-loops, etc.

(2) P was a four place array representing the probability of each issue priority group occurring, i.e., P(1) = probability of issue priority group one occurring, P(2) = probability of issue priority group two occurring, etc.

(3) C, D, M, and V were all four place arrays (inputs to the program) that represented respectively the unit cost, average monthly demand rate, and mean and variance of the order-ship time distribution for each item.

(4) SD was a four place array representing the standard deviation of the order-ship time distribution (componentwise, the SD array was the square root of the V array).

(5) ICC, OC, and TC were four place arrays that represented respectively the inventory carrying cost, ordering cost and total cost of each item in the inventory.

(6) AI and LF were four place arrays used in the calculation of inventory carrying costs. They represented the average inventory and the time of the last issue or receipt of an item.

(7) ITEM and ORDER were both 4 by 360 arrays used to store demands and order arrivals for each item and for each day.

(8) BO, TBO, and DEMAND were all 4 by 4 arrays used respectively to accumulate current backorders by item and issue priority group, accumulate total backorders by item and issue priority group, and accumulate total demands by item and issue priority group.

(9) OH and OO were both four place arrays used respectively to give the current on hand balance by item and current amount on order by item.

(10) ORD, Q, and TD were all four place arrays used respectively to calculate total orders by item, economic order quantity by item, and total demand by item.

(11) ROP was a 4 by 4 array (inputs to the program) that represented the inventory operating levels by item and issue priority group.

APPENDIX C

CHI-SQUARE TEST OF PSEUDORANDOM NUMBER GENERATOR

Each of the integers from 0 to 50 were used as the seed for the multiplicative congruential random number generator used on lines 19 to 29 of the computer program. The unit interval (the range of the pseudorandom number generator) was divided into 100 equal intervals of .01 each. The first 100,000 random numbers in each sequence were generated and the number in each interval of .01 was observed. Then the Chi-Square statistic, $\chi^2_{99} = \sum_{i=1}^{100} \frac{(0_i - 1000)^2}{1000}$, was calculated for each different random number seed (0_i was the observed frequency in the i^{th} interval and 1000 was the expected frequency in each interval). The results for each of the integers from 0 to 50 were as follows:

SEED	CHI-SQUARE	SEED	CHI-SQUARE
0	2.198	16	1927.262
1	2.176	17	2.162
2	30.830	18	31.984
3	2.410	19	2.130
4	59.521	20	60.321
5	2.164	21	2.286
6	31.436	22	32.864
7	2.406	23	2.144
8	174.155	24	174.515
9	2.446	25	2.392
10	31.624	26	31.382
11	2.016	27	2.884
12	59.921	28	59.517
13	2.254	29	2.272
14	32.036	30	32.000
15	2.656	31	2.328

SEED	CHI-SQUARE	SEED	CHI-SQUARE
32	4981.269	42	31.478
33	2.234	43	2.114
34	31.454	44	60.002
35	2.234	45	2.534
36	59.587	46	32.022
37	2.168	47	2.436
38	31.064	48	1926.846
39	2.652	49	2.190
40	174.709	50	31.978
41	2.760		

This supported the conclusion that odd numbers should be used as seeds for the pseudorandom number generator, which was done for all the simulation runs.

APPENDIX D

FURTHER USES OF THE COMPUTER SIMULATION

Many further uses could be made of the computer simulation of the priority inventory model by making relatively minor changes to the program and/or the input data to the program. As mentioned previously, the simulation could be set to run for more or less than 10 years by changing the number 10 on lines 105 and 294 to the desired number of years. This would be particularly useful in decreasing the amount of computer time if that was desired. When the simulation was set to run for 10 years, a set of 77 different inventory operating levels took an average of slightly less than 7 minutes of computer time (approximately $5\frac{1}{2}$ seconds for each set of inventory operating levels). Also, any number of different sets of inventory operating levels could be run by changing the 200 on line 86 to the appropriate value.

The beginning on hand inventory levels of the items in the inventory could be changed by substituting the desired beginning on hand quantity for the 20 on line 88. Each item could have a different beginning on hand inventory level by removing the card on line 88 and substituting the following cards, where A, B, C, and D are the desired beginning inventory levels of each item in the simulated inventory.

```
OH(1) = A;
OH(2) = B;
OH(3) = C;
OH(4) = D;
```

Line 301 could be changed to reflect different empirical probabilities of each issue priority group occurring. Also, lines 302 through 305 could be easily changed to study items with other demand rates (line 302), different order-ship time means (line 303) and variances (line 304) and unit costs (line 305).

More extensive changes could be made to study more than four items at one time, but they will not be mentioned as they involve redefining most of the arrays of variables that are declared on lines two through nine and changing many of the do-loops in the program. However, less than four items could be studied by changing the range of some of the do-loops in the program. If two or three items are desired in the simulated inventory system, change the number 4 on lines 108 and 116 to either two or three, as appropriate. If it is desired to study only one item, change lines 108 and 116 to read

```
I = 1;
```

and remove the END cards on lines 114 and 219. In either of the above cases, the input data should be changed to reflect the number of items in the simulated inventory and the output for those items not in the inventory should be ignored.

The recommended way to study one item is to set all four values of the D array to the average monthly demand rate, set all four values of the M and V arrays to the mean and variance of the order-ship time distribution, and set all four values of the C array to the unit cost of

the item. In effect, the inventory will consist of four identical items and the inventory operating levels could be varied to find the solution to the constrained inventory problem.

By setting inventory operating levels two, three and four equal to zero, the priority inventory model degenerates into the lot size-reorder point model with no explicit backorders costs. Also, minor additions to the program could be made to handle backorder costs. The following are the steps needed to specialize the computer simulation program to handle explicit backorder costs in the lot size-reorder point model.

(1) Let the four place array PI represent the backorder costs per unit backordered (not time-weighted) for each item in the inventory.

(2) Add the following card after line 9.

```
DECLARE (PI(4), ETA(4)) FLOAT BINARY (21);
```

(3) Add the following cards after line 81 (assuming the backorder cost is equal to the unit cost -- if not, substitute the desired backorder cost for each item).

```
ETA(1) = 0.;
ETA(2) = 0.;
ETA(3) = 0.;
ETA(4) = 0.;
PI(1) = C(1);
PI(2) = C(2);
PI(3) = C(3);
PI(4) = C(4);
```

(4) Replace the card on line 84 by the following card;

```
Q(I)=TRUNC(.5+SQRT((120.*D(I)/C(I))*(7.5+(PI(I)*ETA(I)))));
```

(5) Add the following cards after line 226.

```
X = 0.;
Y = 0.;
DO J = 1 TO 4;
```

```
X = X + (PI(I) * TBO(I,J) );
Y = Y + TBO(I,J);
END;
ETA(I) = Y / ORD(I);
```

(6) Replace the card on line 229 by the following card.

```
TC(I) = ICC(I) + OC(I) + X;
```

(7) Add the following cards after line 296.

```
PUT SKIP (2);
PUT EDIT ('AVERAGE NUMBER OF BACKORDERS PER
          PERIOD BY ITEM')(A);
PUT SKIP;
PUT LIST ( ( ETA(I)  DO I = 1 TO 4 ) );
```

(8) Set all the inventory operating levels equal to zero for all items.

(9) Iterate on inventory operating level one (the reorder point) for all items in the inventory until a minimum cost solution is obtained for each individual item (in some cases, it may be necessary to iterate using negative reorder points, also).

(10) Change the values of ETA(1) through ETA(4) (added in step (3) above) to those values printed out at the bottom of the page corresponding to the minimum cost solution for each item.

(11) Repeat steps (8) through (10) until only small changes in the minimum cost solution for each item are obtained in each iteration.

To incorporate multiple demands in a single supply transaction, the following steps would have to be taken:

(1) Remove the card on line 177 and add the following three cards after line 176, where X represents the function that would determine the size of the requisition.

```
QUAN = X;
DEMAND(I,N) = DEMAND(I,N) + QUAN;
DO WHILE ( QUAN = 0 );
```

(2) Add the following two cards after line 216.

```
QUAN = QUAN - 1;
END;
```

APPENDIX E

PARAMETERS USED IN THE SAMPLE PROBLEM

The values of the parameters used in the computer simulation of the priority inventory model were obtained from the Sixth U. S. Army Stock Control Center at the Presidio of San Francisco, California, and are summarized below:

	Item 1	Item 2	Item 3	Item 4
Unit Cost	C (1)=$8.19	C (2)=$.05	C (3)=$11.10	C (4)=$1.33
Average Monthly Demand Rate	D (1) = 3.3	D (2) = 5.7	D (3) = 10.0	D (4) = 2.1
Mean of the Order-ship time Distribution	M (1)=33.5	M (2)=25.5	M (3) = 10.4	M (4)=21.2
Variance of the Order-ship time Distribution	V (1) = 8.8	V (2) = 7.6	V (3) = 1.7	V (4) = 7.8

In addition, probabilities of .04, .1, .56, and .3 were used as the empirical probabilities of issue priority groups 1, 2, 3, and 4 occurring respectively.

The constraints on issue priority groups 1, 2, 3, and 4 respectively were .01, .05, .1, and .2.

```
BEGINNING INVENTORY LEVELS BY ITEM
          20               20               20               20

INVENTORY OPERATING LEVELS BY ITEM (ROWS) AND PRIORITY (COLUMNS)
           0                0                0                0
           0                0                0                0
           0                0                0                0
           0                0                0                0

PROBABILITY OF PRIORITIES 1 THROUGH 4 OCCURING
     3.999999E-02     9.999996E-02     5.599999E-01     2.999999E-01

TOTAL DEMAND BY ITEM
         398              706             1146              271

TOTAL DEMANDS BY ITEM (ROWS) AND PRIORITY (COLUMNS)
          18               29              241                0
          31               66              386              223
          42              107              623              374
           7               33              148               83

TOTAL BACKORDERS BY ITEM (ROWS) AND PRIORITY (COLUMNS)
           2               10               49               19
           0                1                5                3
           5               10               75               33
           2                0                8                1

TOTAL ORDERS BY ITEM
          20                3               41                7

ECONOMIC ORDER QUANTITY BY ITEM
          19              320               28               38

AVERAGE INVENTORY BY ITEM
     6.657046E+02     1.566222E+02     1.166695E+01     1.666732E+01

INVENTORY CARRYING COSTS BY ITEM
     1.090424E+02     1.566221E+01     2.590061E+02     4.433509E+01

ORDERING COSTS BY ITEM
     1.500000E+02     2.250000E+01     3.075000E+02     5.250000E+01

TOTAL INVENTORY COSTS BY ITEM
     2.590422E+02     3.816221E+01     5.665061E+02     9.683509E+01

TOTAL COST OF OPERATING INVENTORY SYSTEM FOR 10 YEARS
     9.605454E+02
```

```
BEGINNING INVENTORY LEVELS BY ITEM
                 20                 20                 20                 20

INVENTORY OPERATING LEVELS BY ITEM (ROWS) AND PRIORITY (COLUMNS)
                  2                  0                  0                  0
                -15                  0                  0                  0
                  2                  0                  0                  0
                 -1                  0                  0                  0

PROBABILITY OF PRIORITIES 1 THROUGH 4 OCCURING
       3.999999E-02       9.999996E-02       5.599999E-01       2.999999E-01

TOTAL DEMAND BY ITEM
                397                690               1248                269

TOTAL DEMANDS BY ITEM (ROWS) AND PRIORITY (COLUMNS)
                 15                 43                220                119
                 28                 68                397                197
                 51                110                721                366
                 14                 29                155                 71

TOTAL BACKORDERS BY ITEM (ROWS) AND PRIORITY (COLUMNS)
                  0                  4                 17                  9
                  4                  7                 30                 18
                  4                  4                 30                 21
                  0                  1                 13                  3

TOTAL ORDERS BY ITEM
                 20                  3                 44                  7

ECONOMIC ORDER QUANTITY BY ITEM
                 19                320                 28                 38

AVERAGE INVENTORY BY ITEM
       8.801731E+00       1.331445E+01       1.310880E+01       1.607012E+01

INVENTORY CARRYING COSTS BY ITEM
       1.441722E+02       1.331445E+01       2.910153E+02       4.274652E+01

ORDERING COSTS BY ITEM
       1.500000E+02       2.250000E+01       3.300000E+02       5.250000E+01

TOTAL INVENTORY COSTS BY ITEM
       2.941721E+02       3.581443E+01       6.210153E+02       9.524652E+01

TOTAL COST OF OPERATING INVENTORY SYSTEM FOR 10 YEARS
       1.046248E+03
```

```
BEGINNING INVENTORY LEVELS BY ITEM
             20                20                20                20

INVENTORY OPERATING LEVELS BY ITEM (ROWS) AND PRIORITY (COLUMNS)
              2                 2                 1                 0
            -15                 4                 1                 0
              2                 4                 1                 0
             -1                 3                 1                 0

PROBABILITY OF PRIORITIES 1 THROUGH 4 OCCURING
      3.999999E-02      9.999996E-02      5.599999E-01      2.999999E-01

TOTAL DEMAND BY ITEM
            395               681              1212               263

TOTAL DEMANDS BY ITEM (ROWS) AND PRIORITY (COLUMNS)
             16                40               210               119
             29                69               385               198
             54               124               663               371
             15                20               142                86

TOTAL BACKORDERS BY ITEM (ROWS) AND PRIORITY (COLUMNS)
              0                 0                22                17
              1                 3                23                16
              1                 1                39                70
              0                 1                12                12

TOTAL ORDERS BY ITEM
             20                 3                43                 7

ECONOMIC ORDER QUANTITY BY ITEM
             19               320                28                38

AVERAGE INVENTORY BY ITEM
      9.118686E+00      1.363872E+01      1.330993E+01      1.666871E+01

INVENTORY CARRYING COSTS BY ITEM
      1.493640E+02      1.363871E+01      2.954802E+02      4.433879E+01

ORDERING COSTS BY ITEM
      1.500000E+02      2.250000E+01      3.225000E+02      5.250000E+01

TOTAL INVENTORY COSTS BY ITEM
      2.993640E+02      3.613871E+01      6.179802E+02      9.683879E+01

TOTAL COST OF OPERATING INVENTORY SYSTEM FOR 10 YEARS
      1.050321E+03
```

```
INV:  PROCEDURE OPTIONS (MAIN);
      DECLARE (A, B, X, Y, Z, COST) FLOAT BINARY (21);
      DECLARE (C(4), P(4), D(4), M(4), V(4), SD(4), TC(4)) FLOAT BINARY (21);
      DECLARE (AI(4), ICC(4), OC(4)) FLOAT BINARY (21);
      DECLARE (H, I, J, K, L, N, SEED) FIXED BINARY (31);            /* LINE 5 */
      DECLARE (ITEM(4,360), ORDER(4,360)) FIXED BINARY (31);
      DECLARE (ORD(4), TRO(4,4), LF(4), OO(4)) FIXED BINARY (31);
      DECLARE (OH(4), BO(4,4), DEMAND(4,4), ROP(4,4)) FIXED BINARY (31);
      DECLARE (Q(4), TD(4)) FIXED BINARY (31);
GET   DATA (SEED, P, D, M, V, C);                                      /* LINE 10 */
A =   RANDOM (SEED);
      /*********************************************************************/
      /* RANDOM RETURNS A PSEUDO-RANDOM NUMBER ON THE INTERVAL 0 TO 1.     */
      /* IF S=0 THEN THE NEXT VALUE IN THE SEQUENCE IS TAKEN.  IF S IS A   */
      /* POSITIVE INTEGER THEN THE SEQUENCE STARTS AT A VALUE DETERMINED   */
      /* BY S.   IF S IS NEGATIVE THEN THE SEQUENCE STARTS AGAIN (RETURNS  */
      /* THE FIRST VALUE GIVEN BY THE INITIAL CALL:  RANDOM (0)).          */
      /*********************************************************************/
      DECLARE RANDOM ENTRY (FIXED BINARY (31)) RETURNS (FLOAT BINARY (21));
      RANDOM:  PROCEDURE (S) FLOAT BINARY (21);                        /* LINE 20 */
         DECLARE (S,RV STATIC INITIAL (3587)) FIXED BINARY (31),
         R FLOAT BINARY (21) STATIC;
         IF S ¬= 0 THEN
            IF S < 0 THEN RV = 3587;
               ELSE RV = S;                                            /* LINE 25 */
         RV = MOD (RV*3587,524288);
         R = RV;
         RETURN (MOD (R,32768)/32768);
      END RANDOM;
      /*********************************************************************/
      /*********************************************************************/
      /* EXPON (S) RETURNS A PSEUDO-RANDOM NUMBER FROM AN EXPONENTIAL      */
      /* DISTRIBUTION WITH A MEAN OF 30/S DAYS.                            */
      /*********************************************************************/
      DECLARE EXPON ENTRY (FLOAT BINARY (21)) RETURNS (FIXED BINARY (31));
      EXPON:  PROCEDURE (S) FIXED BINARY (31);
         DECLARE (S,T) FLOAT BINARY (21);
            T = ( -30 / S ) * ( LOG ( RANDOM (0) ) );
            RETURN ( TRUNC (T + .5));
      END EXPON;                                                       /* LINE 40 */
      /*********************************************************************/
      /*********************************************************************/
      /* NORMAL (M, S) RETURNS A PSEUDO-RANDOM NUMBER FROM A NORMAL        */
      /* DISTRIBUTION WITH A MEAN OF M AND A STANDARD DEVIATION OF S.      */
      /*********************************************************************/
      DECLARE NORMAL ENTRY (FLOAT BINARY (21), FLOAT BINARY (21)) RETURNS
      (FIXED BINARY (31));
      NORMAL:  PROCEDURE (M, S) FIXED BINARY (31);
```

```
     DECLARE (M, N, S, SUM INITIAL (0.0) ) FLOAT BINARY (21);
          DO L = 1 TO 12;                                             /* LINE 50 */
          SUM = SUM + RANDOM (0);
          END;
          N = S * ( SUM - 6 );
          RETURN ( TRUNC ( N + M + .5 ) );
     END NORMAL;                                                      /* LINE 55 */
     /************************************************************************/
     /************************************************************************/
     /* PRIORITY RETURNS A PRIORITY OF 1, 2, 3, OR 4 BASED ON THE           */
     /* EMPIRICAL PROBABILITY OF PRIORITIES 1 THROUGH 4 OCCURING.           */
     /************************************************************************/
     DECLARE PRIORITY ENTRY RETURNS (FIXED BINARY (31));
     PRIORITY:  PROCEDURE FIXED BINARY (31);
         DECLARE C FIXED BINARY (31), A FLOAT BINARY (21);
          A = RANDOM (0);
          IF ( A <= P(1) )  THEN C = 1;                               /* LINE 65 */
              ELSE IF ( A <= P(1) + P(2) )  THEN C = 2;
               ELSE IF ( A <= P(1) + P(2) + P(3) )  THEN C = 3;
                       ELSE C = 4;
          RETURN (C);
     END PRIORITY;                                                    /* LINE 70 */
     /************************************************************************/
     /************************************************************************/
     /* IP(X) CALCULATES THE INVENTORY POSITION OF ITEM X.                  */
     /************************************************************************/
     DECLARE IP ENTRY (FIXED BINARY (31)) RETURNS (FIXED BINARY (31));
     IP:  PROCEDURE (X) FIXED BINARY (31);
         DECLARE (X,Y) FIXED BINARY (31);
          Y = OH(X) + OO(X) - BO(X,1) - BO(X,2) - BO(X,3) - BO(X,4);
          RETURN (Y);
     END IP;                                                          /* LINE 80 */
     /************************************************************************/
DO I = 1 TO 4;
SD(I) = SQRT ( V(I) );
Q(I) = TRUNC ( .5 + ( 30. * ( SQRT ( D(I) / C(I) ) ) ) );
END;                                                                  /* LINE 85 */
DO H = 1 TO 200;
GET DATA (ROP);
OH(*) = 20;
PUT EDIT ('BEGINNING INVENTORY LEVELS BY ITEM') (A);
PUT SKIP;                                                             /* LINE 90 */
PUT LIST ( ( OH(I)  DO I = 1 TO 4 ) );
PUT SKIP (2);
TD(*) = 0;
TC(*) = 0.;
AI(*) = 0.;                                                           /* LINE 95 */
ICC(*) = 0.;
```

```
OC(*) = 0.;
ORD(*) = 0;
TBO(*,*) = 0;
DEMAND(*,*) = 0;                                               /* LINE 100 */
COST = 0.;
ORDER(*,*) = 0;
OO(*) = 0;
BO(*,*) = 0;
DO K = 1 TO 10;                                                /* LINE 105 */
LF(*) = 0;
ITEM(*,*) = 0;
DO I = 1 TO 4;
J = 1 + EXPON ( D(I) );
    DO WHILE ( J <= 360 );                                     /* LINE 110 */
    ITEM(I,J) = ITEM(I,J) + 1;
    J = J + EXPON ( D(I) );
    END;
END;
DO J = 1 TO 360;                                               /* LINE 115 */
DO I = 1 TO 4;
IF ORDER(I,J) > 0 THEN
    DO;
    A = ( J - LF(I) ) * ( OH(I) );
    AI(I) = AI(I) + ( A / 360. );                              /* LINE 120 */
    LF(I) = J;
    OH(I) = OH(I) + ORDER(I,J);
    OO(I) = OO(I) - ORDER(I,J);
    IF OH(I) >= BO(I,1) THEN
        DO;                                                    /* LINE 125 */
        OH(I) = OH(I) - BO(I,1);
        TBO(I,1) = TBO(I,1) + BO(I,1);
        BO(I,1) = 0;
        END;
        ELSE                                                   /* LINE 130 */
            DO;
            TBO(I,1) = TBO(I,1) + OH(I);
            BO(I,1) = BO(I,1) - OH(I);
            OH(I) = 0;
            END;                                               /* LINE 135 */
    IF OH(I) >= BO(I,2) THEN
        DO;
        OH(I) = OH(I) - BO(I,2);
        TBO(I,2) = TBO(I,2) + BO(I,2);
        BO(I,2) = 0;                                           /* LINE 140 */
        END;
        ELSE
            DO;
            TBO(I,2) = TBO(I,2) + OH(I);
```

```
            BO(I,2) = BO(I,2) - OH(I);                                  /* LINE 145 */
            OH(I) = 0;
            END;
      IF OH(I) >= BO(I,3) THEN
         DO;
         OH(I) = OH(I) - BO(I,3);                                       /* LINE 150 */
         TBO(I,3) = TBO(I,3) + BO(I,3);
         BO(I,3) = 0;
         END;
         ELSE
            DO;                                                         /* LINE 155 */
            TBO(I,3) = TBO(I,3) + OH(I);
            BO(I,3) = BO(I,3) - OH(I);
            OH(I) = 0;
            END;
      IF OH(I) >= BO(I,4) THEN                                          /* LINE 160 */
         DO;
         OH(I) = OH(I) - BO(I,4);
         TBO(I,4) = TBO(I,4) + BO(I,4);
         BO(I,4) = 0;
         END;                                                           /* LINE 165 */
         ELSE
            DO;
            TBO(I,4) = TBO(I,4) + OH(I);
            BO(I,4) = BO(I,4) - OH(I);
            OH(I) = 0;                                                  /* LINE 170 */
            END;
      ORDER(I,J) = 0;
      END;
   IF ITEM(I,J) > 0  THEN
      DO WHILE ( ITEM(I,J) > 0 );                                       /* LINE 175 */
      N = PRIORITY;
      DEMAND(I,N) = DEMAND(I,N) + 1;
      IF OH(I) > ROP(I,2) THEN
         DO;
         A = ( J - LF(I) ) * ( OH(I) );                                 /* LINE 180 */
         AI(I) = AI(I) + ( A / 360. );
         OH(I) = OH(I) - 1;
         LF(I) = J;
         END;
         ELSE IF ( OH(I) > ROP(I,3) & N < 4 ) THEN                      /* LINE 185 */
            DO;
            A = ( J - LF(I) ) * ( OH(I) );
            AI(I) = AI(I) + ( A / 360. );
            OH(I) = OH(I) - 1;
            LF(I) = J;                                                  /* LINE 190 */
            END;
            ELSE IF N = 4 THEN BO(I,N) = BO(I,N) + 1;
```

```
            ELSE IF ( OH(I) > ROP(I,4) & N < 3 ) THEN
                DO:
                A = ( J - LF(I) ) * ( OH(I) ):                    /* LINE 195 */
                AI(I) = AI(I) + ( A / 360. ):
                OH(I) = OH(I) - 1:
                LF(I) = J:
                END:
                ELSE IF N >= 3 THEN BO(I,N) = BO(I,N) + 1:        /* LINE 200 */
                    ELSE IF ( OH(I) > 0 & N = 1 ) THEN
                        DO:
                        A = ( J - LF(I) ) * ( OH(I) ):
                        AI(I) = AI(I) + ( A / 360. ):
                        OH(I) = OH(I) - 1:                        /* LINE 205 */
                        LF(I) = J:
                        END:
                        ELSE BO(I,N) = BO(I,N) + 1:
    IF IP(I) <= ROP(I,1) THEN
        DO:                                                       /* LINE 210 */
        L = NORMAL ( M(I), SD(I) ):
        IF J + L > 360 THEN ORDER(I,J+L-360) = Q(I):
            ELSE ORDER(I,J+L) = Q(I);
        OO(I) = OO(I) + Q(I):
        ORD(I) = ORD(I) + 1:                                      /* LINE 215 */
        END;
    ITEM(I,J) = ITEM(I,J) - 1:
    END;
END:
END:                                                              /* LINE 220 */
DO I = 1 TO 4:
A = ( 360 - LF(I) ) * ( OH(I) ):
AI(I) = AI(I) + ( A / 360. ):
END:
END:                                                              /* LINE 225 */
DO I = 1 TO 4:
ICC(I) = ( .2 ) * ( AI(I) ) * ( C(I) ):
OC(I) = ( 7.5 ) * ( ORD(I) ):
TC(I) = ICC(I) + OC(I):
COST = COST + TC(I):                                              /* LINE 230 */
END:
DO I = 1 TO 4:
DO N = 1 TO 4;
TD(I) = TD(I) + DEMAND(I,N):
END:                                                              /* LINE 235 */
END:
PUT EDIT ('INVENTORY OPERATING LEVELS BY ITEM (ROWS) AND PRIORITY (COLUMNS)')
(A):
PUT SKIP:
DO I = 1 TO 4:                                                    /* LINE 240 */
```

```
PUT LIST ( ( ROP(I,J)  DO J = 1 TO 4 ) );
PUT SKIP;
END;
PUT SKIP;
PUT EDIT ('PROBABILITY OF PRIORITIES 1 THROUGH 4 OCCURING') (A);  /* LINE 245 */
PUT SKIP;
PUT LIST ( ( P(I)  DO I = 1 TO 4 ) );
PUT SKIP (2);
PUT EDIT ('TOTAL DEMAND BY ITEM') (A);
PUT SKIP;                                                         /* LINE 250 */
PUT LIST ( ( TD(I)  DO I = 1 TO 4 ) );
PUT SKIP (2);
PUT EDIT ('TOTAL DEMANDS BY ITEM (ROWS) AND PRIORITY (COLUMNS)') (A);
PUT SKIP;
DO I = 1 TO 4;                                                    /* LINE 255 */
PUT LIST ( ( DEMAND(I,J)  DO J = 1 TO 4 ) );
PUT SKIP;
END;
PUT SKIP;
PUT EDIT ('TOTAL BACKORDERS BY ITEM (ROWS) AND PRIORITY (COLUMNS)') (A);
PUT SKIP;
DO I = 1 TO 4;
PUT LIST ( ( TBO(I,J)  DO J = 1 TO 4 ) );
PUT SKIP;
END;                                                              /* LINE 265 */
PUT SKIP;
PUT EDIT ('TOTAL ORDERS BY ITEM') (A);
PUT SKIP;
PUT LIST ( ( ORD(I)  DO I = 1 TO 4 ) );
PUT SKIP (2);                                                     /* LINE 270 */
PUT EDIT ('ECONOMIC ORDER QUANTITY BY ITEM') (A);
PUT SKIP;
PUT LIST ( ( Q(I)  DO I = 1 TO 4 ) );
PUT SKIP (2);
PUT EDIT ('AVERAGE INVENTORY BY ITEM') (A);                       /* LINE 275 */
PUT SKIP;
DO I = 1 TO 4;
AI(I) = AI(I) / 10.;
PUT LIST ( AI(I) );
END;                                                              /* LINE 280 */
PUT SKIP (2);
PUT EDIT ('INVENTORY CARRYING COSTS BY ITEM') (A);
PUT SKIP;
PUT LIST ( ( ICC(I)  DO I = 1 TO 4 ) );
PUT SKIP (2);                                                     /* LINE 285 */
PUT EDIT ('ORDERING COSTS BY ITEM') (A);
PUT SKIP;
PUT LIST ( ( OC(I)  DO I = 1 TO 4 ) );
```

```
PUT SKIP (2);
PUT EDIT ('TOTAL INVENTORY COSTS BY ITEM') (A);                        /* LINE 290 */
PUT SKIP;
PUT LIST ( ( TC(I)  DO I = 1 TO 4 ) );
PUT SKIP (2);
PUT EDIT ('TOTAL COST OF OPERATING INVENTORY SYSTEM FOR 10 YEARS') (A);
PUT SKIP;                                                              /* LINE 295 */
PUT LIST (COST);
PUT PAGE;
END;
END INV;                                                        /* END OF PROGRAM */
SEED = 3,                                     /* START OF DATA */  /* LINE 300 */
P(1) = .04,            P(2) = .1,             P(3) = .56,            P(4) = .3,
D(1) = 3.3,            D(2) = 5.7,            D(3) = 10.0,           D(4) = 2.1,
M(1) = 33.5,           M(2) = 25.5,           M(3) = 10.4,           M(4) = 27.3,
V(1) = 8.8,            V(2) = 7.6,            V(3) = 1.7,            V(4) = 10.1,
C(1) = 8.19,           C(2) = .05,            C(3) = 11.1,           C(4) = 1.33;
ROP(1,1) = 0,          ROP(2,1) = 0,          ROP(3,1) = 0,          ROP(4,1) = 0,
ROP(1,2) = 0,          ROP(2,2) = 0,          ROP(3,2) = 0,          ROP(4,2) = 0,
ROP(1,3) = 0,          ROP(2,3) = 0,          ROP(3,3) = 0,          ROP(4,3) = 0,
ROP(1,4) = 0,          ROP(2,4) = 0,          ROP(3,4) = 0,          ROP(4,4) = 0;
ROP(1,1) = 2,          ROP(2,1) = -15,        ROP(3,1) = 2,          ROP(4,1) = -1;
ROP(1,2) = 2,          ROP(2,2) = 4,          ROP(3,2) = 4,          ROP(4,2) = 3,
ROP(1,3) = 1,          ROP(2,3) = 1,          ROP(3,3) = 1,          ROP(4,3) = 1;
```

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13. ABSTRACT

Very few inventory models of the military supply system have been developed that explicitly take into account the effects of the priority system. The priority inventory model of the military supply system is formulated to minimize the sum of ordering and holding costs subject to constraints on the number of backorders allowable by issue priority group. A minimum cost solution of a typical inventory problem of this nature is obtained by computer simulation.

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